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GEOMETRIC–OPTICAL ILLUSIONS: A PEDESTRIAN'S VIEW OF THE PHENOMENAL LANDSCAPE

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Abstract

In the present paper some types of geometric–optical illusions are discussed, with a particular focus on distortions of shape in contour fields. A "principle of contiguous variation" is proposed as a heuristic tool in search for a unitary description, i.e., a truly phenomenological theory of these phenomena, and some consequences of the proposal for experimental research are pondered.

> "It is not customary to speak of comparative physics in the same sense that we speak of comparative anatomy. ... But like all other sciences, physics lives and grows by comparison." — Ernst Mach [15]

Geometric–optical illusions (GOI) are context-dependent visual distortions of extent or shape of simple geometrical figures, usually linear drawings, discovered some 150 years ago and intensively studied since then. Nowadays it is commonly agreed that perceptual illusions in general, and GOIs in particular, are not random errors or marginal anomalies of sensory systems but rather lawful manifestations of their functional principles [17]. Still, no unified theory of GOIs exists, there is no commonly accepted explanation and no consensus about general principles upon which such an explanation may rely: a puzzling and frustrating situation indeed.

The aim of the present paper is not to advocate any of the current explanatory approaches or theories. Instead, adopting a functional view of theory [21], we propose a strategy possibly leading to a working phenomenological theory of GOIs.

Mapping the phenomenal landscape

Theory, $\vartheta\varepsilon\omega\rho\iota\alpha$, means originally "beholding", "contemplation." Hence our metaphor of a theory of a phenomenal field as of watching and exploring a landscape. In the landscape there are some prominent, visited and named points, separated by large, rarely visited spaces. These are open to various exploratory strategies. A researcher may ascend a single hill repeatedly, study its geography and geology, and try to understand the surrounding land by generalization of the local findings. Or he may adopt a bird's eye perspective and design a large-scale map of the land, in an attempt of a unifying and ultimate "explanation." Instead of those "bottom-up" climbing or "top-down" watching approaches, we advocate a "pedestrian" approach, studying the landscape by really traversing it and mapping the ways connecting its different regions. Then *the proper methodology of exploring the phenomenal landscape is a study of its "hodology," of its system of paths.* Metrization of the field, a coordination system put over the landscape, comes only later, as the next step.

We will illustrate the "pedestrian" approach to theory on one particular subclass of GOIs, namely, shape distortions induced by superimposed contour fields. Nonetheless, the ideas presented here may be relevant for the study of other perceptual phenomena as well.

Principle of contiguous variations

Consider the well-known Hering illusion [12]: a straight line drawn over a bundle of lines meeting in one point appears slightly bent (Fig. 1a).[1] For simplicity, we call the part of the figure, on which a distortion is observed, the *target* stimulus, and the additional components of the figure
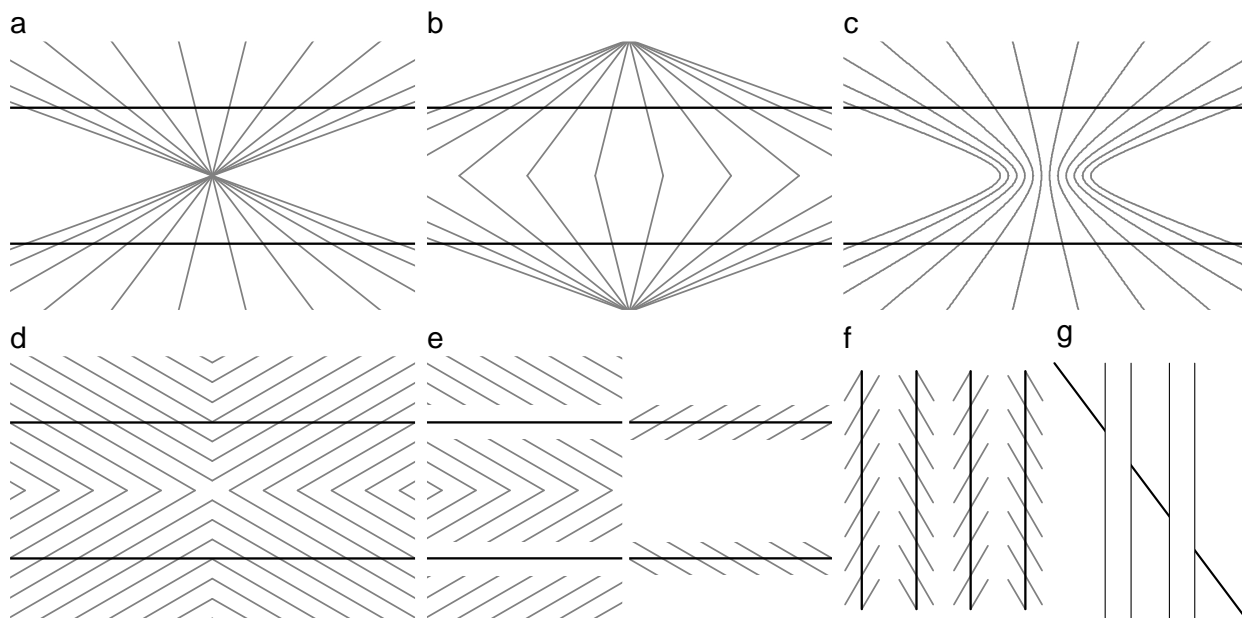
Figure 1. (a) Hering curvature; (b) Wundt's variant of Hering's figure; (c) Hering curvature induced by hyperbolic context; (d) 'Broken lines' in an array of parallels; (e) Effects of partial context erasure: left, parallelism is restored, right, illusory convergence persists; (f) Zöllner tilt; (g) Poggendorf shift.

the *context* stimulus. In Wundt's variant [20, 24] of Hering illusion the upper and lower part of the context pattern are swapped; a similar curvature of the target lines in the opposite direction is seen (Fig. 1b). Hering's figure can be further modified; *e.g.* in Fig. 1c the context consists of an array of hyperboles intersecting the target lines and a curvature comparable to Fig. 1a is observed: a concentric structure of the context pattern is thus not necessary for the illusory effect to occur.[2]

In Fig. 1d, the context pattern is composed of straight lines crossing two parallels in equi-spaced points, at angles $\pm 30°$; again, the target lines appear symmetrically bent or 'broken'. Fig. 1e demonstrates that the curvature effect results from *local* interactions between the target and the context, independently from the context's overall structure. The two parallel line segments in Fig. 1e right seemingly converge; cutting out the hatched stripes, replicating them and rotating by $90°$ yields the Zöllner illusion [25] of tilted verticals (Fig. 1f). A closer look at Figs. 1e,f eventually reveals that the oblique context lines appear broken and slightly shifted by the "passage" through the target line—a reciprocal effect of the target elements on the context elements, observable also in Figs. 1a–d. This is a limiting case of the Poggendorf illusion (Fig. 1g) for line-drawn figures.[3]

The relations among the three GOIs are, of course, nothing new; they were repeatedly pointed out since their early discoveries. The point of this brief demonstration is that the three groups of phenomena (Hering curvature, Zöllner tilt, and Poggendorf shift) are *connected* by series of subsequent, *contiguous variations*. It is the whole system of transformations—or, say with Wittgenstein, their "grammar"—what defines the class of studied phenomena in its entirety.

Intrinsic vs. extrinsic variation

A controlled variation of a parameter of the studied system, while keeping other parameters constant or undetermined, is the leading method of all experimentation [16]. As far as the phenomenon under study does not change qualitatively within the variation limits, we may call this procedure *intrinsic* variation. Most of empirical findings on GOIs have been gathered using this method. — Closely related is the method of "eidetic variation," used by some authors to isolate a generating principle of some GOIs in a thought experiment, or to restrict the number of varied parameters in a real laboratory experiment. Procedures based on this version of intrinsic variation were properly dubbed "amputations" and "perturbations" [23]; it is often questionable if effects isolated and investigated by this method are identical with the original phenomenon of interest.

By contrast, the principle of contiguous variation outlined above aims at the connections between *apparently different* phenomena: it could be denoted the method of *extrinsic* variation. Surely, this approach reminds more of the method of comparative biology or linguistics than of physical sciences. At the first look it seems that it does not bring out anything more than a classification of phenomena; but this criticism has to be critically examined.

Classifications vs. explanations

If science is about establishing an order in a multitude of phenomena, then *classification* is the first step toward this goal. In common understanding this is only a provisional, transitory stage: classification is seen as merely descriptive knowledge, to be sooner or later superseded by *explanations* derived from underlying principles and mechanisms (theory-building). Accordingly, most authors of reviews on GOIs reduce the role of classificatory schemes to mere "convenience" [14], or to an "taxonomic exercise [which] does not itself provide explanations" [19, p. 20]. Other authors, however, feel the lack of a satisfactory typology of GOIs as really troubling: "It is true that classification does not in and of itself provide us with explanations. Nonetheless, many advances in the sciences have been triggered by the creation of a meaningful classification system" [5, pp. 200–201]. The authors named Linné's biological taxonomy or Mendeleyev's periodical system of chemical elements as paradigmatic examples.

This is a more differentiated attitude towards classification,[5] yet not radical enough. Abandoning the worn-off *cliché* "classification is not explanation," and seeing realistically: explanation is nothing but a successful subsumption of a phenomenon under a known regularity; and a working theory is nothing but a system of applicable regularities ("rules", "laws"). From this functional point of view [21], the relational structure induced by a classification is a theory of its own merit.[6] The conception of theory as a successfull classificatory scheme—and, correlatively, the subordination of explanation to classification—possibly alien to a modern reader, is not new, and by no way unheard. It was elaborated in the beginning of the 20th century by the French physicist Pierre Duhem, who distinguished between the *representative* and *explanatory* components of a theory:

> Everything good in the theory, by virtue of which it appears as a natural classification and confers on it the power to anticipate experience, is found in the representative part; all of that was discovered by the physicist while he forgot about the search for explanation. [7, p. 32]

It is the representative part which guarantees continuity of scientific knowledge, while the explanatory part is always fragile, and often subject to modifications or revisions [7, *loc. cit.*].

Psychophysics, conceived by Fechner as a science of *functional relations* [10], leaves the burden of explanations via mediating mechanisms to neighboring disciplines, and aims naturally at a phenomenological—that is, purely representative—theory as its ideal.

Consequences and corollaries

Context transformations

The term "classification" used above suggests too easily a system of discrete classes or labeled lists. A more adequate notion is that of a *continuous manifold*: a suitable parametrization of the phenomenal domain lets the classificatory boundaries dissolve in a "continuum of facts" [15]. In our special case, a "point" in this continuum is a vector of parameters, determining the geometry and density of the contour field used as the context pattern.[7] The target elements (lines, arcs, circles) can be modeled in the same way; preferably, both targets and contexts would be generated by the same parametric system, so that mutual target–context effects can be also studied.

Perceptual distortions are thus modeled by mappings between parametric spaces on which the contour fields are defined. Of special interest are limiting cases shared by two contour field systems; *e.g.*, an array of parallel straight lines is a limiting case of a pencil of lines (such as in Fig. 1a) as well as of a system of concentric circles (such as in Fig. 2a), with centers escaping to infinity, and
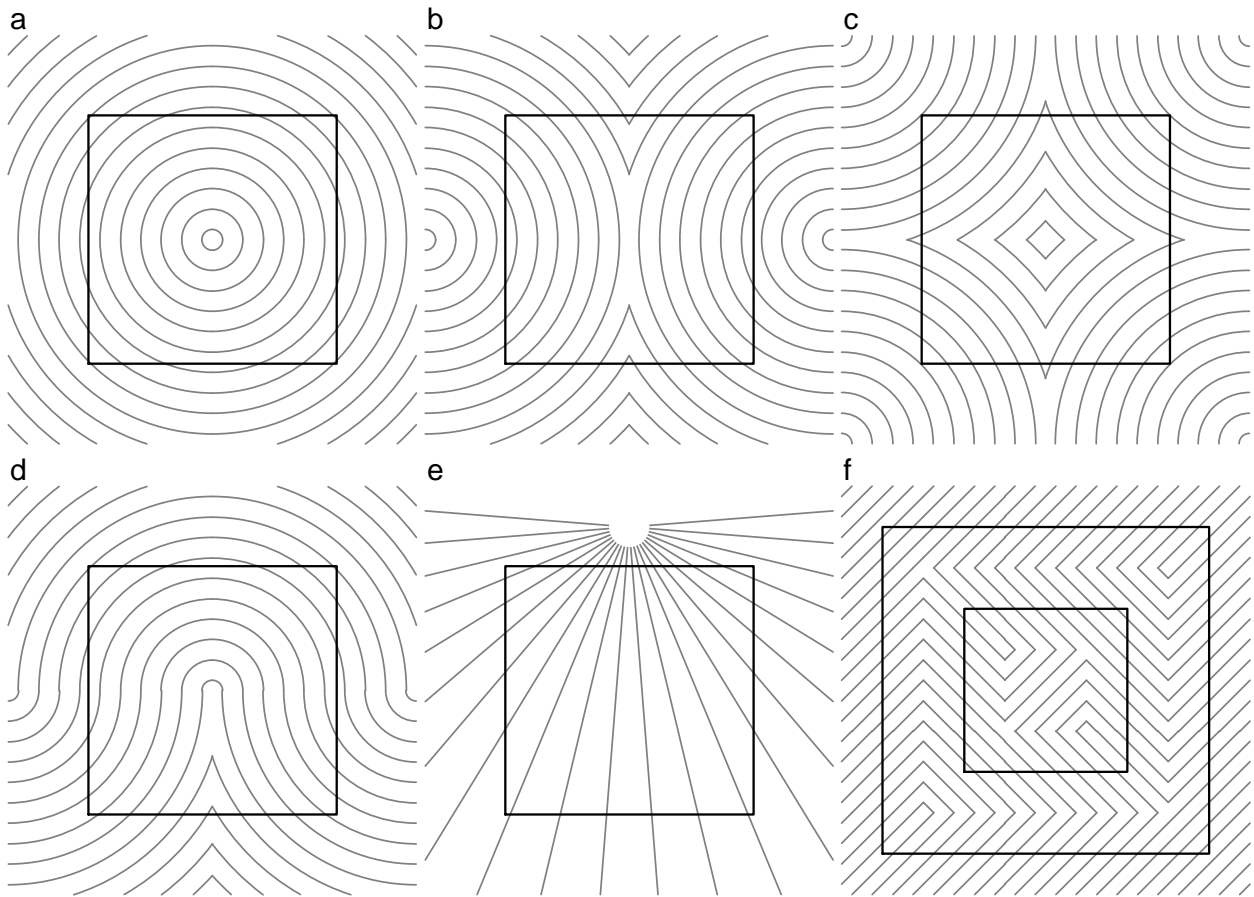
Figure 2. Distortions of a square shape induced by different contexts. (a) "Cushion" deformation induced by an array of concentric cirles; (b) "Barrel" deformation in a modified context; (c) minimal deformation in a further modified context; (d) "Trapezoid" deformation in a context composed of circular arcs ("ocean wave"), and (e) similar deformation induced by a bundle of concentric lines; (f) "Rhomboid" deformation of squares in a tiled array of parallel lines.

spacing/density parameters properly adjusted. Such limiting cases establish "junctures" between contour field systems, connecting apparently different types of GOIs. Intrinsic variations act along continuous paths in the parametric space; extrinsic variations are mediated by these "junctures."

A useful parametrization should also provide a simple representation of *discontinuous* context transformations (*e.g.* cutting/pasting or mirror symmetry), providing further insight into the structure of the studied phenomenon. For example, the Ehrenstein–Orbison illusion [8, 18] (Fig. 2a) is inverted in Fig. 2b, or abolished in Fig. 2c by partial permutation of the context pattern.

Arguments for local interactions

Particularly important are instances of "multiple realization": identical or similar distortion of a target figure in different contexts, such as the "trapezoid" deformation of a square in Fig. 2d,e. These are convincing counter-examples against intepretations of GOIs in terms of fictitious depth cues and "unconscious inferences" drawn from them.[8] The global, "scenic" impression is arguably irrelevant for the effect (compare also Fig. 1a,c). The observed distortions are thus due to *local* target–context interactions, and probably reducible to Zöllner-like alterations of perceived angles of intersection (*cf.* Fig. 2f).[9]

Cross-context measurements

Psychophysical experiments are often understood as measurements of subjective sensations. In our view, measurements are made in the objective world, and the subject's role consists solely in establishing perceptual equivalences between world-states [22]. Illusory percepts are not directly measured. One cannot say that radius of the curvature in Fig. 1a is 40 cm; one can only compare

a GOI percept against another percept of a context-free stimulus, *e.g.* a circular arc of a known (physically measurable) radius. Or one can use distortion induced in the same target by a different context as a measurement referent; in other words, one can study perceptual equivalences between different variants or types of GOIs. No study utilizing this interesting option is known to us.

Conclusion

The charm and challenge of geometric–optical illusions (GOI) consists in the fundamental character of the problem they raise: the subjective metric of the visual space, and the dependence of the metric on the visual content; ultimately, the emergence of metrical notions from the matter of primary experience. Psychophysics should address this problem directly, without explanatory terms borrowed from neighboring disciplines (neurophysiology, cognitive psychology, etc.)

Looking back at 150 years of research into GOIs, two major tendencies can be recognized: (1) extensive experimental studies aiming at a possibly exhaustive description of a singular phenomenon, and (2) hasty attempts of general theories—or rather conjectures raised to the status of universal explanatory principles. While the former is harmless, the latter is demonstrably harmful to the progress of knowledge. Too often were GOIs employed as "*experimenta crucis* for the theories" [1, p. 239] instead of studied for their own sake. The present paper should be understood as an invitation to a rediscovery journey through the landscape of perceptual phenomena; the principle of contiguous variations gives a guide for the journey, but not a ready-made map.[10]

Notes

[1] Fig. 1a is an approximation of Hering's original Fig. 25 in [12, p. 74]. Many variants of the figure are known, differing in the numerosity and spacing of the context lines, distance between the target lines, and orientation of the whole figure. Modern sources generally prefer equi-angular line bundles and vertical orientation of the parallels; *e.g.* [3, 9, 13, 19]. Duplicity of target lines is *not* essential for the illusion; it is observed in a single line as well.

[2] The hyperboles are constructed so that their tangents at the points of intersection with the target lines meet in *two* different points in the central line. Incidentally, this observation invalidates explanations of this and related phenomena from "depth cues" or "imagined movement" [3], based on a 3-dimensional perceptual interpretation. An unprejudiced look at Figs. 1a–d does not support any such "unconscious inferences" [11]: a perspectival interpretation is questionable in Fig. 1a, unlikely in Figs. 1b,c, and plainly impossible in Fig. 1d (showing a distortion similar to Fig. 1a), which appears perfectly flat.

[3] Or, more precisely, a limiting case of Poggendorf illusion with the stripe-width reduced to the drawing line width. This illusion was indeed discovered as a side effect of Zöllner's illusion [25].

[4] Practiced with astonishing virtuosity by Brentano [2] in his interpretations of Müller-Lyer illusion.

[5] To understand the quote in its specific context: Coren and Girgus argued for a multicausal approach to GOIs, so that the intended classification should reflect this causal multiplicity. Our approach sketched in the following is essentially different from their naïvely multivariate statistics-based "taxonomy" [4].

[6] To emphasize the latter point: Mendeleyev's periodic system is more than "just a table"; it *is* a (kind of) theory of chemical elements, and it really *does* meet expectations imposed on a theory, including its potential to make successful predictions (discoveries of yet unknown elements).

[7] Consider a real-valued *field form* function f (sufficiently smooth and "well behaved") defined in the drawing plane $\mathbb{R}^2$, and a monotonic real-valued *pick-up* function h defined on $\mathbb{Z}$. The locus of points $\ell_n = \{(x_1, x_2) | f(x_1, x_2) = h(n)\}$ defines a (curvi)linear element in the drawing plane; the enumerable system $F = \{\ell_n | n \in \mathbb{Z}\}$ is a unique contour field. Making the field form function dependent on a parameter vector $\vartheta \in \Theta \subseteq \mathbb{R}^m$, we obtain a parameterized system of contour fields $\mathfrak{F}\langle f_\vartheta, h, \Theta \rangle$.

[8] *Cf.* Gregory's [9] and Day's [6] explanatory theories based on "size constancy" scaling.

[9] Studies of GOI effects under angle-preserving spatial tranformations, such as the circle inversion, would be certainly interesting.

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References

[1] Boring, E. G. (1942). *Sensation and Perception in the History of Experimental Psychology.* New York: Appleton, Century, & Crofts.

[2] Brentano, F. (1892) Über ein optisches Paradoxon. *Zeitschrift für Psychologie und Physiologie der Sinnesorgane*, *3*, 349–358.

[3] Changizi, M. A., Hsieh, A., Nijhawan, R., Kanai, R., & Shimojo, S. (2008). Perceiving the present and a systematization of illusions. *Cognitive Science*, *32*, 459–503.

[4] Coren, S., Girgus, J. S., Erlichman, H., & Hakstian, A. R. (1976). An empirical taxonomy of visual illusions. *Perception and Psychophysics*, *20*, 129–137.

[5] Coren, S. & Girgus, J. S. (1978). *Seeing is Deceiving: The Psychology of Visual Illusions.* Hillsdale (NJ): Lawrence Erlbaum.

[6] Day, R. H. (1972). Visual spatial illusions: a general explanation. *Science*, *175*, 1335–1340.

[7] Duhem, P. (1954). *The Aim and Structure of Physical Theory*. Princeton: Princeton University Press. [Original French title, *La Théorie Physique, Son Objet, Sa Structure*, published 1902.]

[8] Ehrenstein, W. (1925). Versuche über die Beziehungen zwischen Bewegungs- und Gestaltwahrnehmung. *Zeitschrift für Psychologie*, *96*, 305–352.

[9] Gregory, R. L. (1968). Perceptual illusions and brain models. *Proceedings of the Royal Society*, *B171*, 279–296.

[10] Heidelberger, M. (2010). Functional relations and causality in Fechner and Mach. *Philosophical Psychology*, *23*, 163–172.

[11] Helmholtz, H. (1867). *Handbuch der physiologischen Optik.* Leipzig: Voss.

[12] Hering, E. (1861). *Beiträge zur Physiologie I. Vom Ortsinne der Netzhaut.* Leipzig: Engelmann.

[13] Holt-Hansen, K. (1961). Hering's illusion. *British Journal of Psychology*, *52*, 317–321.

[14] Luckiesh, M. (1922). *Visual Illusions: Their Causes, Characteristics and Applications.* New York: Van Nostrand.

[15] Mach, E. (1898). On the principle of comparison in physics. In E. Mach, *Popular Scientific Lectures* (transl. by Th. J. Cormack), 3rd ed., Chicago: Open Court.

[16] Mach, E. (1926). *Erkenntnis und Irrtum.* 5th ed., Leipzig: Barth.

[17] Metzger, W. (1975). *Gesetze des Sehens.* 3rd ed., Frankfurt: Kramer.

[18] Orbison, W. D. (1939). Shape as a function of the vector field. *American Journal of Psychology*, *52*, 31–45.

[19] Robinson, J. O. (1998). *The Psychology of Visual Illusion.* 2nd ed., Mineola (NY): Dover.

[20] Thiéry, A. (1895) Ueber geometrisch-optischen Täuschungen. *Philosophische Studien*, *11*, 307–370, 603–620; *12*, 67–126.

[21] Wackermann J. (2006). Rationality, universality, and individuality in a functional conception of theory. *International Journal of Psychophysiology*, *62*, 411–426.

[22] Wackermann, J. (2010). Psychophysics as a science of primary experience. *Philosophical Psychology*, *23*, 189–206.

[23] Weintraub, D. J. & Krantz, D. H. (1971). The Poggendorf illusion: Amputations, rotations, and other perturbations. *Perception and Psychophysics*, *10*, 257–264.

[24] Wundt, W. (1898) Die geometrisch–optischen Täuschungen. *Abhandlungen der mathematisch-physischen Classe der Königl. Sächsischen Gesellschaft der Wissenschaften*, *24*, 55–178.

[25] Zöllner, F. (1860). Ueber eine neue Art von Pseudoskopie und ihre Beziehungen zu den von Plateau und Oppel beschriebenen Bewegungsphänomenen. *Annalen der Physik und Chemie 186*, 500–523.