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Beyond mapping: estimating complexity of multichannel EEG recordings

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Abstract. A new measure of complexity of multichannel EEG recordings is proposed. The quantity assesses the diversity of activities of different brain generators contributing to the global dynamics of electric field of brain.

Key words: multichannel EEG complexity, spatio-temporal dynamics of brain activity, spatial principal components analysis, sleep EEG

INTRODUCTION

There are two major strategies of quantitative analysis of multichannel EEG recordings. The first, time-oriented strategy attempts to characterize the bioelectric signals recorded at separate locations by traditional methods of analysis in time-domain. The alternative, space-oriented strategy aims at non-ambiguous description of entire potential field distribution at a time, using interpolated potential maps and various space-domain descriptors (see Lehmann 1987).

The space-oriented approach is superior to the time-oriented one as it does not neglect correlations between recording locations (electrode sites). A series of maps is a direct and, in fact, the most "natural" representation of the global spatio-temporal field dynamics. On the other hand, the space-domain analysis still has to pose and solve many questions; concepts of stationarity, segmentation, and adequate parametrization have to be rethought and eventually reformulated for global field distributions.

Space-oriented analysis can benefit from a well-defined notion of state space. Basic concepts of this approach are briefly presented here; then it is shown how the state space representation yields a measure of complexity of map series.

STATE-SPACE REPRESENTATION OF MAP SERIES

Consider a multichannel EEG record based on simultaneous measurements at K electrode sites ($K>1$). A snapshot of brain's electric field at time t is represented by a K-dimensional vector $u(t)$. An entire time epoch measured at a given sampling frequency is then represented by a sequence of N voltagevectors $\{u(t_n)\}_{n=1...N}$. These data vectors - momentary electric states - can be interpreted geometrically as points of an (abstract) K-dimensional linear vector space (see Naylor and Sell 1971, for an introduction), a state space. The time evolution of the brain's electric states is then represented by a sequence of points, a trajectory in the state space.

The concepts of scalar product and norm of a vector, are defined generally for a K-dimensional space; this allow us to measure distances and angles in the state space. For instance, the measure of global field power of a map, used in the space-oriented analysis (Lehmann 1987), is obviously equivalent to the Euclidean norm of the corresponding data vector (up to a multiplicative constant).

In the theory of vector spaces, the metric of the state space is generated by a function of two vector arguments, called scalar product u * v. Generally, this product can be defined by $u * v = u^T G v$, where G is an appropriate symmetrical matrix. Here we are dealing with the simplest case of a space with Euclidean metric only, so that the matrix G is the unit matrix. If, however, we took the assumption of continuous field distribution into account, we would have to replace the unit matrix by a more general form,where the non-diagonal elements of matrix G corresponding to neighboring electrode locations are non-zero. In the following sections, we will intentionally disregard this correction for field continuity.

The semantics of the state space is thus quite straightforward; there are one-to-one correspondences between the traditional language of "brain mapping" and the state space representation. Table I summarizes these correspondences in a form of a "dictionary".

TABLE I

Correspondences between basic concepts of brain mapping and state space representation

Brain mapping	State space
field distribution (map)	point
zero field	origin of coordinates (0)
field landscape	direction
base map	coordinate axis
global field power	distance from the origin
map dissimilarity	distance between points
map correlation	angle between directions

SPATIAL PCA

An EEG trajectory in the state space makes up a "data cloud" of maps (data vectors) snapped at discrete time intervals. Now we want to find an orthonormal system of base vectors so that any map (data vector) can be expressed as a linear combination of orthonormal base vectors:

$$u = s_1 \cdot v_1 + \ldots + s_K \cdot v_K$$

Solving this task, we find that the base vectors can be obtained as eigenvectors of the covariance matrix C. Assuming data being centred over time, the matrix C is defined by

$$c_{ij} = \frac{1}{N} \sum_{n=1}^{N} u_{in}\, u_{jn}$$

The expansion of the matrix C into its eigenvectors $v_1, \ldots, v_K$ with eigenvalues $\lambda_1, \ldots, \lambda_K$, respectively, is as follows:

$$C = \sum_{i=1}^{K} \lambda_i\, v_i\, v_i^T$$

Geometrically, the eigenvalues represent the proportions of variance along these new axes, so that

$$\sum_{i=1}^{K} \lambda_i = tr\, C \equiv \sum_{i=1}^{K} c_{ii} \equiv \text{ total variance} \qquad (1)$$

The solution is, in fact, equivalent to the procedure known as the Principal Component Analysis. Because here we are dealing with covariances between spatially distributed measuring locations, the resulting eigenvectors can be interpreted again as spatial distributions - base maps. This is why we speak about Spatial Principal Component Analysis (SPCA).

A well-known extremal condition holds for the principal components: if were order the eigenvalues in the descending order ($\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_K$), then the first component (v_1) exhausts maximum of the total variance, the second component (v_2) exhausts maximum of the residual variance, etc. This property guarantees that any subspace spanned on M components ($M<K$) with highest λ's represents maximum of the total variance of the data, and therefore provides an optimal projection of the original data cloud into a space of lower dimension (see Rao (1973, section 8g) for a more rigorous treatment).

VISUALIZATION OF EEG TRAJECTORIES

Although the geometrical interpretation of the state space is quite intuitive, it suffers by our lack of ability to visualize a space of more than three dimensions. We can use the results of SPCA to obtain a projection of the K-dimensional state space to a subspace of less dimensions (2-3), taking two or three principal components with highest eigenvalues. Due to the extremal property of principal components stated above, it is ensured that this projection represents maximum of variance.

In this way we obtain the best possible projection of the trajectory into a 2- or 3-dimensional space. Only 2-dimensional projections are suitable for prints; 3-dimensional projections can be inspected by means of 3D data viewers; a lot of such software is available nowadays, e.g. Acrospin or Gnuplot.

Figures 1A - E provide illustrative examples of state space trajectories reconstructed from EEG recorded in different functional brain states. (Keep in mind that the figures display merely a projection to a 2-dimensional space of the two major principal components.). The variety of forms of those trajectories entails a question of suitable characterization of their properties by few quantitative descriptors.

INTRODUCING Ω-COMPLEXITY

SPCA performed on a given data epoch yields a set of eigenvectors and their respective eigenvalues. The eigenvectors represent the topography of the

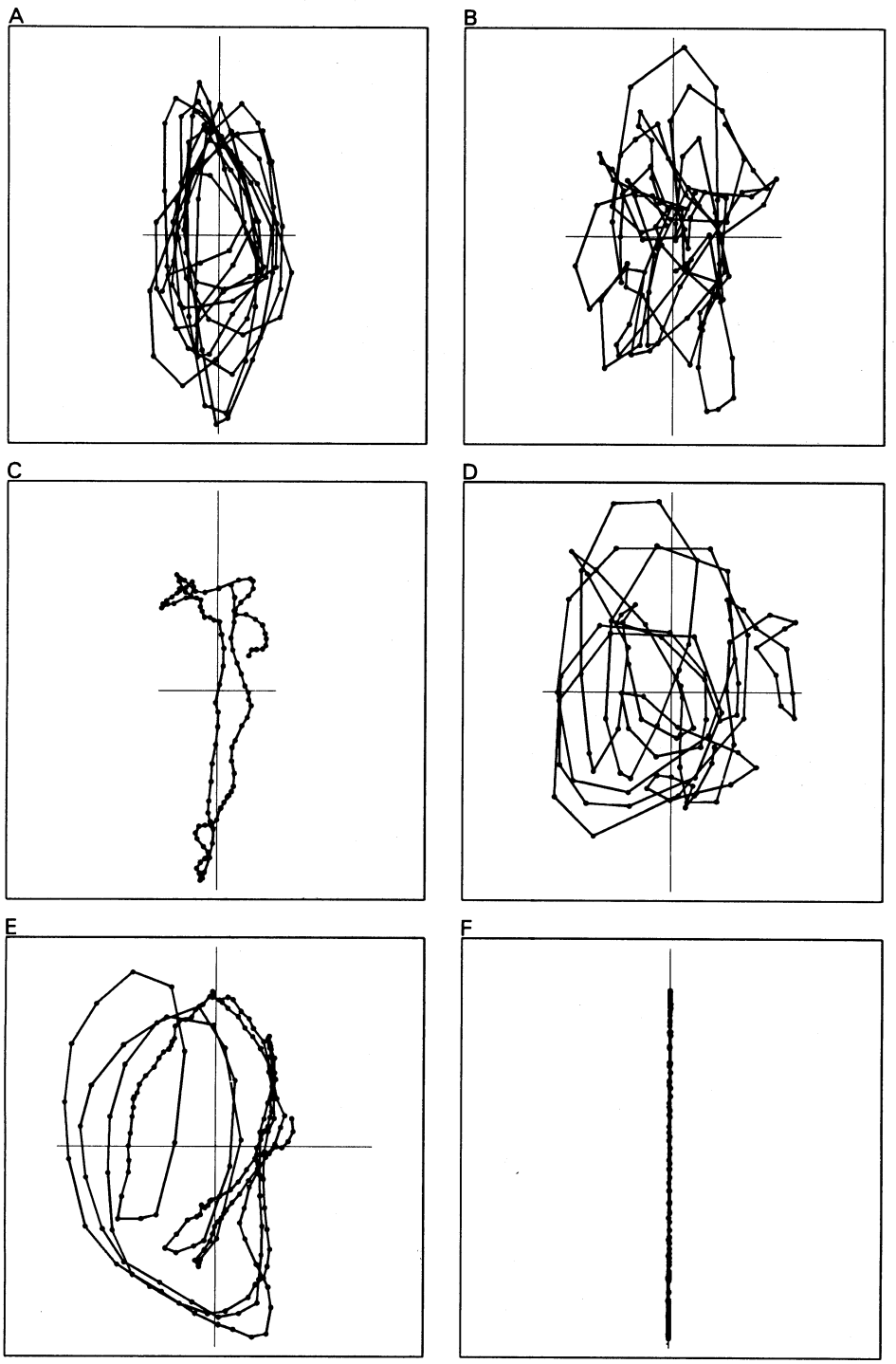

Fig. 1. Examples of 2-dimensional projections of state trajectories of 1 s of spontaneous EEG activity, constructed from 21 or 19 channel recordings (system 10/20). Length of axes proportional to $\sqrt{\lambda_{1,2}}$. Note that sampling frequency f_S differs between examples. A, relaxed waking state with dominant alpha rhythm; K=21, f_S=128/s. B, relaxed waking state, flat fast activity; K=21, f_S=128/s. C, sleep EEG, stage 4, with dominant slow waves; K=21, f_S=102.4/s. D, sleep spindle in stage 2; K=21, f_S=102.4/s. E, paroxysmal spike-wave activity (3/s) in a case of petit mal epilepsy; K=19, f_S=200/s. F, calibration signal (5/s); K=19, f_S=200/s.

base maps; the eigenvalues represent the proportions of contribution of each of these base maps to the total variance. If we are interested in the proportions only, disregarding absolute values, we will normalize the eigenvalues to a unit sum

$$\lambda_i' = \frac{\lambda_i}{T}, \text{ where } T \equiv tr\ C$$

and call the set $\Lambda = \{ \lambda'_1, ..., \lambda'_K \}$ a Λ-spectrum of the covariance matrix (do not confuse with frequency spectrum). Figure 2 displays Λ-spectra computed for the EEG data, trajectories of which were shown on Fig. 1. This figure makes it clear that Ω is a measure of "flatness" of Λ-spectrum.

The form of Λ-spectrum obviously bears an important information on the covariance structure of the data and, consequently, on the hidden generating structures. Let us consider two extreme cases:

1. A single generator of field described by base map b_1. Then b_1 is the eigenvector corresponding to the only non-zero eigenvalue λ_1; all other eigenvalues are zero. Therefore, the matrix C has a "degenerate" Λ-spectrum {1, 0, ..., 0}. (This case is easily modelled by feeding all amplifier inputs by identical signal, as illustrated by Figs. 1F and 2F).

2. K uncorrelated generators with equal power, one for each of the K measuring locations. Then C will be a diagonal matrix with all eigenvalues $\lambda_i = c_{ii}$ equal. Consequently, the matrix C has a uniform Λ-spectrum $\{1/K,...,1/K\}$.

Although these two extreme cases are not physiologically realistic, they provide anchor points for a definition of a measure of complexity. What we need is a function of λ's defined in such a way that its value is 1 for case (1), and K for case (2).

DEFINITION OF Ω

The reasoning sketched briefly above lead us to the following heuristic definition of multichannel

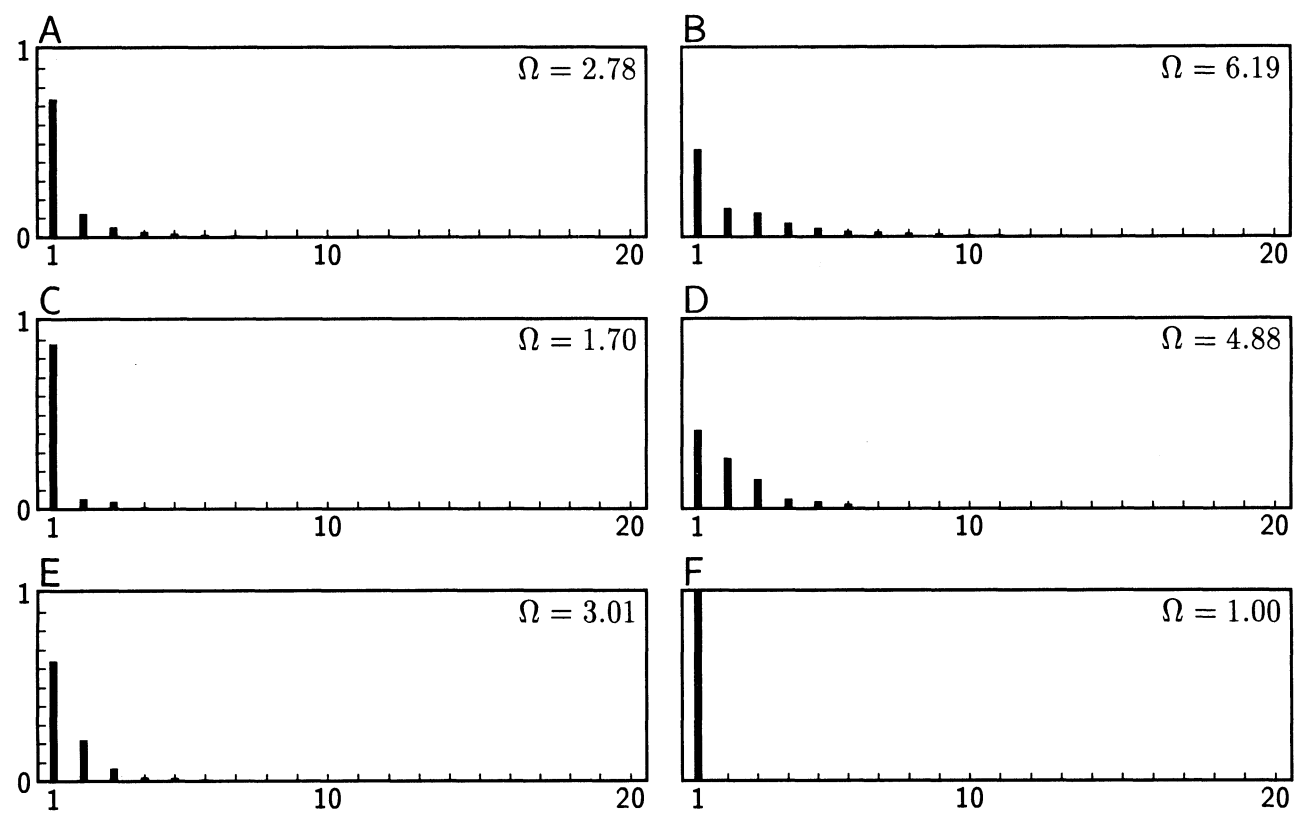

Fig. 2. Λ-spectra and corresponding Ω values computed for the six trajectories shown in Fig. 1A-F.

EEG complexity. Be $\{ \lambda'_1, ..., \lambda'_K \}$ a Λ-spectrum of the covariance matrix of the data. We then define

$$\Omega = \exp\left\{ -\sum_{i=1}^{K} \lambda'_i \log \lambda'_i \right\} \qquad (2)$$

(If one or more eigenvalues are zero, we formally set 0 log 0 = 0.)

PROPERTIES OF Ω

According to the definition given above, Ω has the following properties:

1. Ω is invariant to data scaling. If the data vectors are multiplied by a constant c, the eigenvalues change by factor c^2; this effect will be cancelled by normalization of eigenvalues, so that λ' do not change.

2. Ω is invariant to time scaling. It is obvious from the bare fact that the time dimension disappears due to summation over time index n in computations of covariance matrix *C*. Therefore, Ω depends only on covariance structure of the data, not on absolute frequencies.

3. Ω yields values ranging from 1 to *K*. It can be easily shown that the function $-\Sigma\lambda'_i \log \lambda'_i$ reaches its maximum, log *K*, at $\Lambda=\{1/K,...,1/K\}$. On the other side, the function vanishes in the degenerate case $\Lambda=\{1,0,...,0\}$. The exponential function in the formula (2) merely maps the interval [0, log *K*] to the interval [1, *K*] as required.

4. Ω yields a lower bound estimate of number of generators. If there are *R* uncorrelated generators ($R \leq K$), the Λ-spectrum is

$$\Lambda = \left\{ \underbrace{\frac{1}{R}, ..., \frac{1}{R}}_{R\text{x}}, \underbrace{..., 0, ..., 0}_{(K-R)\text{x}} \right\}, \text{ which implies } \Omega=R$$

GROSS STATES DESCRIPTION

The Ω descriptor, by its very nature, is a parameter of an epoch of certain time extent; at least few hundreds of data samples are required for a reliable estimate of the covariance matrix. Ω is thus no candidate for studies on a time scale of tens or hundreds milliseconds, in the range of brain microstates (Lehmann et al. 1987). The proper area for its use are studies of brain macrostates, gross states of brain functioning, of duration in orders of minutes or longer.

APPLICATION TO SLEEP DATA

In order to examine the behaviour of the new descriptor, we opted for human sleep EEG recordings, as the sleep stages show obvious differences in spatial synchronization of the electrical activity.

A whole-night record of sleep EEG (duration 7:44 h, sampled at f_s = 102.4/ s, system 10/20) was analysed by time slices of 2.5 s. To reduce the time resolution to that used in sleep scoring routine, medians of each eight consecutive Ω values were computed, so that the time profile was constructed of 20 s epochs.

Independently, sleep stages were scored for each of these 20 s epochs according to scoring system by Rechtschaffen and Kales (1968), and artifact counts were determined. Only epochs without artifacts were taken for the subsequent statistics, although

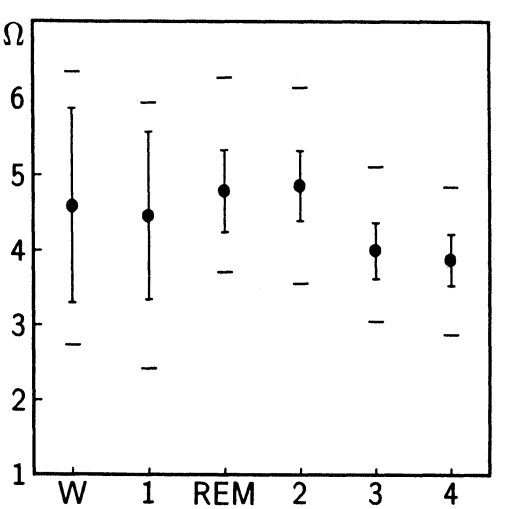

Fig. 3. A comparison of Ω-complexity for distinct sleep stages (average±1 SD and min/max). ANOVA: F=146.48, df=5,884, n=470, P<0.001.

this somewhat reduced the number of accepted epochs.

The results of between-stage comparison are shown on Fig. 3. An ANOVA test has confirmed highly significant differences between sleep stages; note the apparent decrease of complexity during slow wave sleep.

This example is taken from a larger study on differences in Ω-complexity between sleep stages and sleep cycles by Szelenberger et al. (1996).

EXTENDING THE DESCRIPTION

Although our investigations concern mainly the complexity of brain electric activity, we feel that a description of brain in terms of complexity only would be badly incomplete. What we need is a complementary description in terms of average voltage and dominant frequency, or more exactly, multidimensional analogs of these traditional measures.

To this purpose we compute estimates of integral squared norms of state vectors and their differences in time:

$$m_0 = \frac{1}{N}\sum_{n=1}^{N} \| u_n \|^2$$

$$m_1 = \frac{1}{N-1}\sum_{n=1}^{N-1} \| \Delta u_n \|^2$$

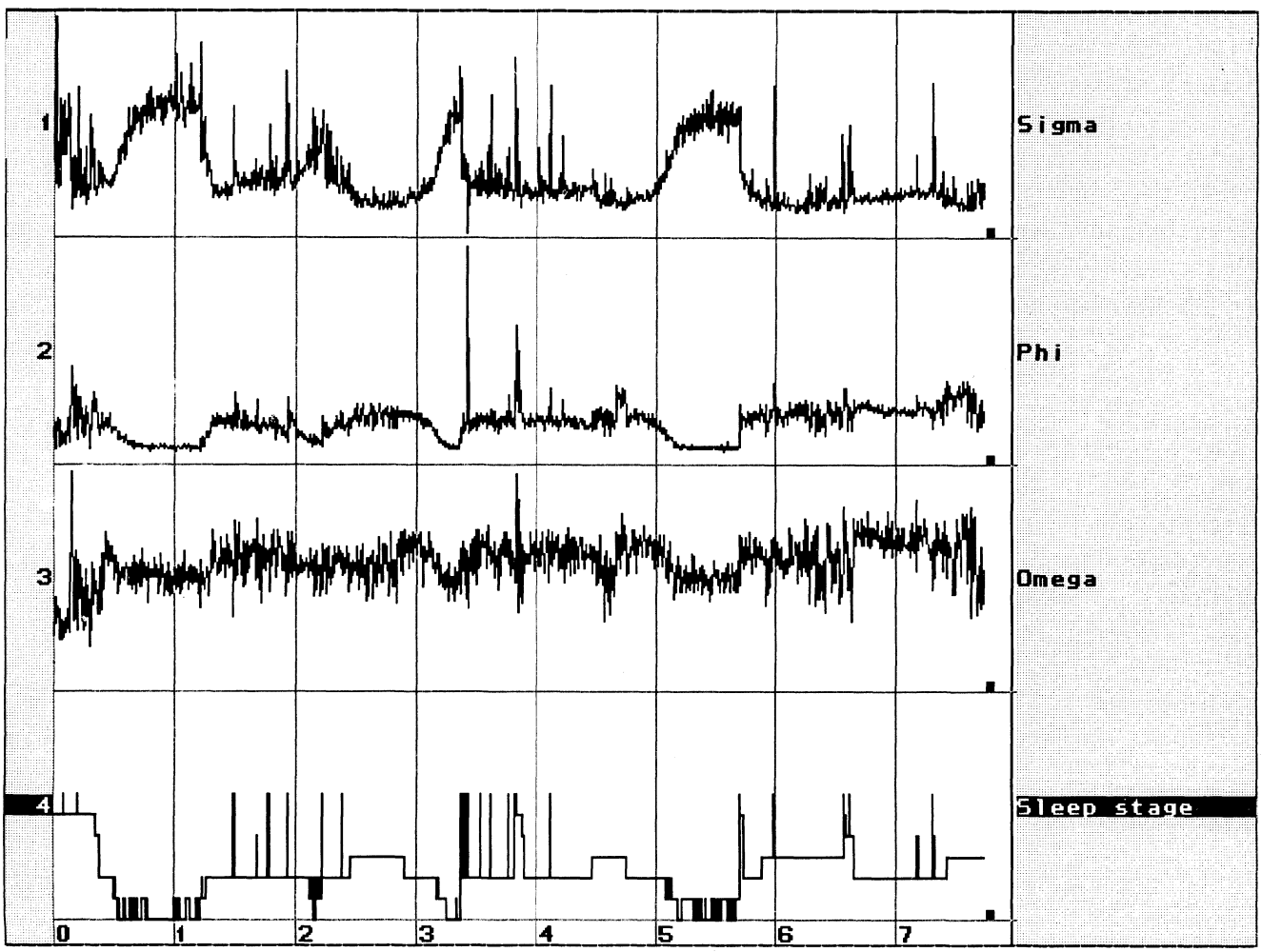

Fig. 4. A synoptic plot of Σ, Φ, and Ω descriptors, computed by 20 secepochs for a whole-night sleep EEG record. Below a profile of visually scored sleep stages, in the order MT, W, S1, REM, S2, S3, S4. Time axis labelled by hours.

Speaking mapping language, m_o is equivalent to the integral of squared global field power, while m_1 is the integral of squared (unnormalized) map dissimilarity between successive maps.

Obviously, m_o is identical to the total variance T in formula (1); thus, the quantity

$$\Sigma = \sqrt{m_o/K} \ [\mu V]$$

(where K = number of channels) represents the effective voltage per channel.

Furthermore, the quantity

$$\Phi = \frac{1}{2\pi\Delta t}\sqrt{m_1/m_o} \ [s^{-1}]$$

(where Δt=sampling step) has the property of generalized frequency; this can be easily shown if we imagine a circular trajectory in the state space which makes Φ windings per second.

The descriptors Σ and Φ are, in fact, multidimensional analogs of Hjorth's descriptors, activity and mobility, respectively (Hjorth 1973).

DYNAMICS IN [Σ, Φ, Ω] SPACE

Using the three descriptors together, we can assign each analysed epoch of an EEG record three coordinates [Σ, Φ, Ω]. Dividing a long-term record into epochs of equal length, we obtain time series of these descriptors; in this way we can assess the dynamics of gross states of the brain over long time periods (Fig. 4); this, of course, at the cost of enormous data reduction.

Again, we can represent each epoch as a point in a "gross state space" anduse a 3D display tools to study the structure of the data cloud. First experiences with such a representation of sleep EEG records in the [Σ, Φ, Ω] space are very encouraging. Marking epochs according their respective sleep stages reveals differences between states (Fig. 5).

Since we use the term "state space" in two different meanings, it is important to avoid confusion of the concepts. The state space discussed above - that is, the space of momentary electric states of the brain - had dimension K (number of electrodes) and its axes were all labelled in voltage units (μV). The gross state space described here is a 3-dimensional space of physically heterogeneous dimensions (μV, Hz, and dimensionless Ω); it is something like the [p, V, T] space of states of a gas volume in elementary physics.

DISCUSSION

Relationships to other EEG measures

Ω is defined in such a way that it is *in principle* independent of the voltage or frequency, as stated above. This, however, does not mean that Ω will not correlate with other EEG measures. Inspection of the results indicates that, generally, data epochs of higher voltage and lower frequencies have lower complexity (see Fig. 4). The relationships are obviously non-linear and rather complex.

In this context it is necessary to distinguish between logical and empirical independence. The former is given *a priori* by a mathematical proof; the latter is given by experimental findings. In a sense, the logical independence is a prerequisite for meaningful investigations of empirical correlations.

Relationships to other methods of analysis

The method of assessment of complexity of multichannel EEG recordings presented here is much closer to space-oriented methods than to classical time-domain techniques, as it works with entire data vectors at a time. On the other hand, it is definitely beyond scope of the traditional mapping approach. Rather than to reduce a map to some "spatial descriptors" in the real 2-dimensional space of electrode array, it operates in an abstract K-dimensional state space without preliminary data reduction.

As for the SPCA, Skrandies and Lehmann (1982) used this technique for statistical processing of evoked potential data, with the aim to find principal components of EP distributions at selected

latency times. Lehmann (1987, p. 325) also lists Spatial PCA in his outline of a system of spatial analysis. In both cases SPCA was treated as a back-end method, used for final statistical data analysis and presentation. In our approach, SPCA serves rather as a front-end method, that is, a technique of transformation of input data.

Generally speaking, SPCA is a special case of projection of observed data into a (sub)space spanned on a appropriately selected orthonormal base. In case of SPCA, the base vectors are determined solely by the data, so this is a truly "data-driven" procedure. Alternatively, a predefined set of orthogonal vectors or spatial functions could be used to the same purpose; this was the approach taken by Fuchs et al. (1987) in their analysis of alpha-EEG.

Paluš (1991) proposed a measure called "linear complexity" which was somewhat similar to Ω; this quantity was, however, a function of eigenvalues of the correlation matrix (which is different from the original covariance matrix) and its construction was different.

Recently, Ziller et al. (1995) have studied performance of Hjorth's time domain descriptors and correlation dimension (D_2) in classification of sleep stages. They found combination of Hjorth's "mobility" and "complexity" to be superior to classifi-

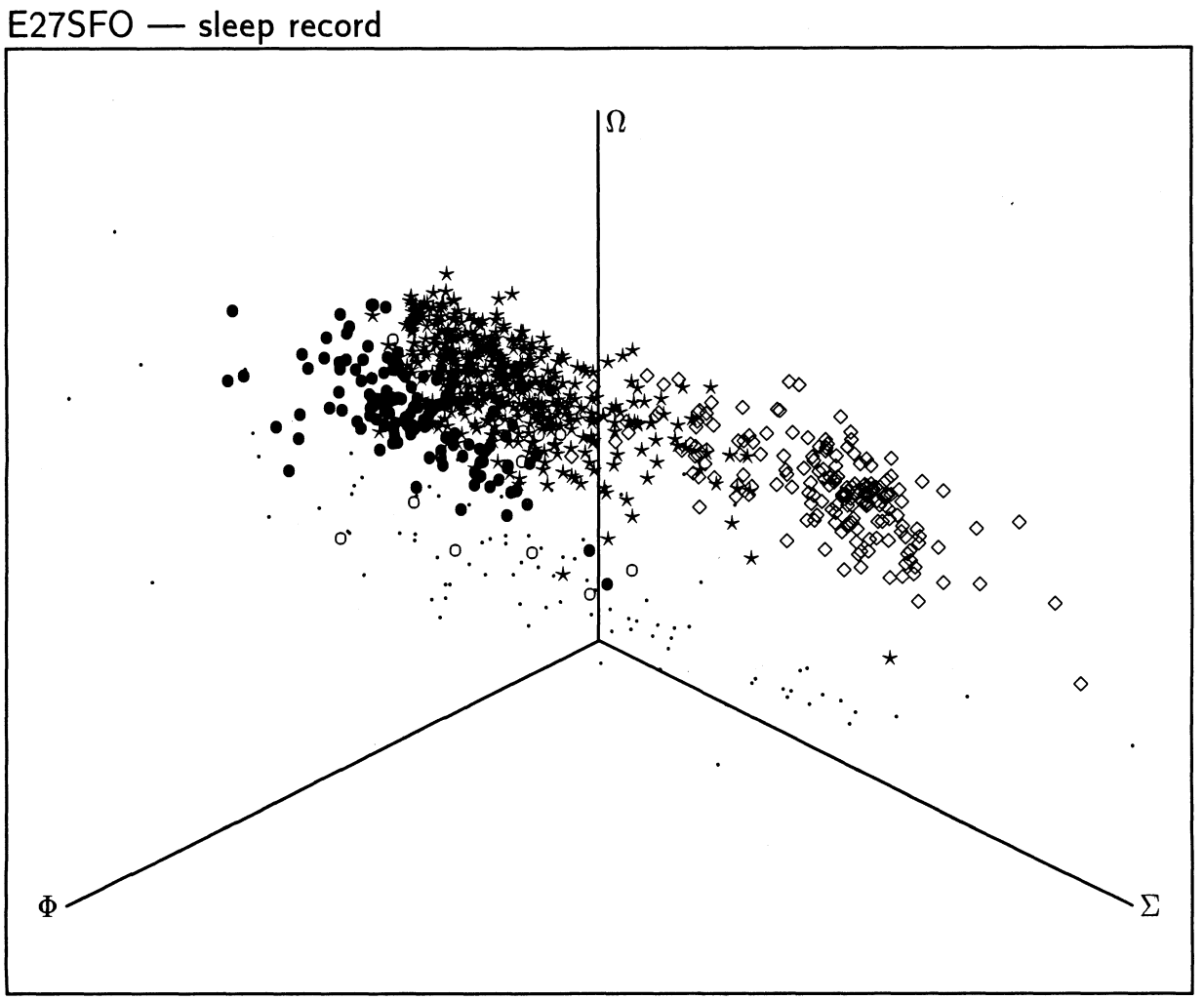

Fig. 5. A portrait of a whole-night sleep EEG record (same as in Fig. 4) in the [Σ, Φ, Ω] space. Axes: Σ 0 - 24 μV, Φ 0 - 12 Hz, Ω 0 - 6. Sleep stages distinguished by graphic symbols: dots, waking; open circles, stage 1; asterisks, REM; filled circles, stage 2; diamonds, stages 3 and 4.

cation by D_2. Their work, although based on one-channel record only, goes to some extent in the same direction as sleep EEG study presented here. However, the final goal is different: Ziller et al. opposed time-domain descriptors in order to find better descriptors, while we take time-domain descriptors and complexity measure together to obtain as complete description as possible.

As mentioned above, the Σ and Φ descriptors are, in fact, generalization of Hjorth's "activity" and "mobility" to a multi-dimensional signal. Hjorth's "complexity", being a measure of dispersion of the frequency spectrum, has nothing to do with Ω-complexity or D_2-complexity. (The term "complexity" is obviously heavily overloaded).

Methodical and technical considerations

There are few technical problems which still have to be solved. Perhaps the most important among them is the length of epoch of analysis. A reliable estimate of the covariance matrix is necessary for computation of Ω. A lower limit for the number of data points should be determined; in praxis, we decided deliberately that the epoch length should be at least 1 s. If we take too short epochs, then the volume covered by the trajectory is not filled densely enough to allow for valid estimate of the covariance matrix; on the other hand, with too long epochs we run the risk of mixing activities of different generator configurations, hence increasing the resulting estimate of complexity. Systematic investigations in this area are required.The same applies to the influence of other factors, for example, number of recording channels (K).

What concerns the details of implementations, these do not affect the essential question of viability of the method. On the other hand, if systematic studies are to be performed, an effective implementation is a must. The most time consuming part of the algorithm is computation of the eigenvalues of the covariance matrix. We currently use the Jacobi diagonalization method (Golub and Van Loan 1983); it is stable and easy to implement, but it is known to be not the fastest. A choice of a more sophisticated algorithm may appear necessary with large volume of data.

Yet another complexity?

The issue of complexity of brain electrical activity is traditionally associated to the theory of non-linear systems - popularised as "chaos" theory - and their mathematical characterization (for reviews, see Rapp et al. (1989) and Pritchard and Duke (1992). Thus, our proposal for a new descriptor of EEG complexity may provoke an uneasy reaction: why, yet another complexity?

It should be stated explicitly that Ω-complexity is not a new breed of "chaos" measures, and was not introduced as a replacement or "improvement" of them. While the latter were designed to characterize systems with strong non-linearities which could not be adequately described by standard methods, the apparatus of Ω-complexity relies on well-known linear methodology. Most of complexity studies concerned one-dimensional time series, using the Takens method of "delayed coordinates" to create a reconstruction of state space trajectory, Ω is by definition a descriptor of multi-dimensional time series.

Nevertheless, implicit relationships exist between Ω-complexity and some applications of "chaos" measures to EEG. Following the conjecture made by Eckman and Ruelle (1985), Dvořák (1990, 1991) proposed a generaliaztion of the correlation dimension to multi-channel EEG recordings covering entire scalp or large portions of the scalp. This "global" dimensional complexity turned out to be a rather sensitive indicator of changes in brain functioning induced by a nootropic drug (Wackermann et al. 1993). Recently, Matoušek et al. (1995) found significant correlation between differences of global dimensional complexity of awake/drowsy EEG, and subjects' age on the other side.

It is true that the generalized, global dimensional complexity and Ω-complexity provide quantitative description of the same object, that is, the EEG trajectory in the K-dimensional state space. Nevertheless, the two measures differ in their mathematical

definition, and in their intent as well. Ω-complexity is really a global quantitative descriptor of EEG trajectory which has an intuitively clear geometrical interpretation. Moreover, Ω is easy to compute and does not involve any technical intricacies similar to those occurring with "chaos" measures.

CONCLUSIONS

Ω complexity proposed in the present paper appears to be a promising descriptor of covariance structure of a multichannel EEG record, and consequently a suitable measure of complexity of spatio-temporal dynamics of the electric activity of the brain.

Preliminary results obtained in a study on human sleep EEG indicate that Ω may truly reflect physiological states of the brain, particularly when embedded in a 3-dimensional [Σ, Φ, Ω] system of EEG descriptors. Further experimental data should clarify detailed relationships between Ω and other parameters, and assess the amount of incremental information beared by the new descriptor.

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